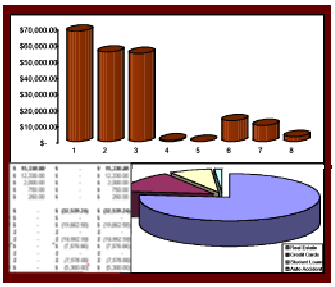


Week 8: Linear Programming

- ✓ Quiz:
  - Linear programming relies on finding optimal solutions for functions of variables with defined constraints. In one sentence or less, write a definition for the words “function,” “variable,” and “constraint.”

- ✓ Quiz for next week:
  - Project management focuses on managing three things: scope, schedule, and budget. In one sentence or less, write a definition for the words “scope,” “schedule,” and “budget” as they relate to a project.

- ✓ Assignments due this week:
  - Biography: Siméon-Denis Poisson
  - Inventory Spreadsheet: Build a spreadsheet that calculates EOQ and TC for a single product.

- ✓ Assignments due next week:
  - Biography: Pythagoras
  - Linear Programming Spreadsheet: Build a spreadsheet that solves a function with at least two variables and two constraints.

- ✓ What is a line? What is a program? What is a linear program?
  - A line is a mathematical expression that can be written in the form $y = mx + b$
  - A program is a system for solving a problem.
  - So linear programming is a way of finding solutions to problems using lines.

- ✓ Objective function: What you are trying to do.
  - Maximize something (usually sales or profits)
  - Minimize something (usually cost or time)

- ✓ Constraints: Your limitations.
  - Greater than (>)…
  - Less than (<)…
  - Equal to (=)…
  - Not equal to (≠)…
  - Greater than or equal to (≥)…
  - Less than or equal to (≤)…

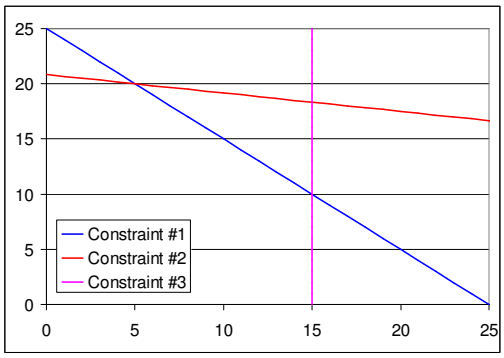
- ✓ Feasible Solutions. Every solution that falls within the constraints is a feasible solution.

- ✓ Optimal Solution. The solution(s) which falls within the constraints and satisfies the objective is the optimal solution.

- ✓ Solution Techniques
  - Graphical: Simple linear programs can be solved graphically.
  - Mathematical: Solving even complex linear programming problems is easy once you learn how to set them up in a spreadsheet.

- ✓ How do you spot a linear programming (LP) problem in the wild?
  - Objective function is linear (no square roots, nothing raised to a power)
  - Constraints are linear (no square roots, nothing raised to a power)

- ✓ Example: Linear Program Graphical Solution
  - Objective: Maximize $x_1 + x_2$
  - Constraints: $x_1 + x_2 \geq 25$
    $2x_1 + 12x_2 \leq 250$
    $x_1 \leq 15$



✓ Example: Linear Program Mathematical Solution

- We have been hired by a local company that manufactures very exclusive custom motorcycles using an exotic titanium alloy. Our client has two product lines, a sport bike and a roadster. Their goal is to make as many motorcycles as they can this year, thus maximizing their profits. However, there are some important factors that they need to consider. First of all, they estimate that they need to make at least 25 bikes in order to cover their overhead expenses. Next, because of strong demand from the military and the aircraft industry they won't be able to acquire any more titanium alloy until next year. They have 250 pounds of titanium in inventory, so they need to make the most of what they have. Each sport bike uses 2 pounds of titanium, and each roadster uses 12 pounds of titanium. Finally, to maintain the exclusivity of their sport bike line, they do not want to make any more than 15 of the sport bikes per year.
- Objective: Maximize number of bikes made ($x_1 + x_2$)
  x_1 : Number of Sport Bikes
  x_2 : Number of Roadsters
- Constraints: Make the minimum to cover overhead ($x_1 + x_2 \geq 25$)
  Use only the titanium we have in inventory ($2x_1 + 12x_2 \leq 250$)
  Maintain exclusivity of sport bikes ($x_1 \leq 15$)
- Note: You cannot make a negative number of either product ($x1 \geq 0$, $x2 \geq 0$). It is often necessary to specify this constraint in computer models. If you get an error that says a solution could not be found, check to see if you need to define numbers that cannot be less than zero.

Linear Programming Example

	Number Made	*Overhead*	*Titanium*	*Exclusivity*
Sport Bikes	15	15	30	**15**
Roadsters	18	18	220	
	33	**33**	**250**	

- ✓ Spreadsheet features that are useful for linear programming
  - Formulae
  - Charting/Graphs
  - Min/Max and other functions
  - Conditional formatting
  - Goal seek
  - Solver (Need to install Solver Add-in in Excel)

- ✓ What do you do with an LP when you find it?
  - "Problem formulation" or "modeling" expresses a verbal statement of the problem in a mathematical form.
    - Understand the problem thoroughly
    - Describe the objective
    - Describe each constraint
    - Define the decision variables
    - Write the objective in terms of the decision variables
    - Write the constraints in terms of the decision variables

- ✓ Building in Slack
  - Sometimes we want to account for resources that are available, but unused. For example, let's say we are manufacturing products A and B, and we have a total of 100 hours of labor available. It takes one hour to make each unit of A, and two hours to make each unit of B. Typically, we would write this constraint this way:

$$(1 \times A) + (2 \times B) = L$$

  - But this approach ignores a key element of the constraint, which is the total labor available. So, we also want to track how much of the available labor we are NOT using… the slack. In this case, the labor constraint equation would look like this:

$$(1 \times A) + (2 \times B) + S = 100$$